

# INTEREST RATE ARBITRAGES

## Part II

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## Detrended Fluctuations

The idea of trading “detrended fluctuations” can be applied not only to single assets but also to relationships among two or more closely related assets.

In the futures markets, the most obvious of these are relationships among the various “tenors” of the yield curve. These are the yields at various times to maturity expressed in the market via the prices of Treasury futures.

The tenors are affected by many economic and market factors. For example, the short end of the curve is driven mostly by monetary policy. The very long end may be driven by inflation expectations while the middle part driven by the amount of corporate issuance.

Nonetheless, the tenors are also locked into a mathematical relationship prescribed by the forward rate curve, whose structure invariably implies that interest rates are mean reverting over time.

The interplay between the disparate factors that drive the tenors and the overall constraint on how much they can move relative to each other gives rise to many mean reversion trading opportunities.

## Level, Slope and Convexity

The movements in the yield curve can be decomposed approximately into changes in *level*, *slope* and *convexity*.

The level refers to the general level of interest rates, which, loosely speaking, is the average yield over all maturities (a intellectually neater definition will be given later).

The slope refers to the difference between long-dated and short-dated yields.

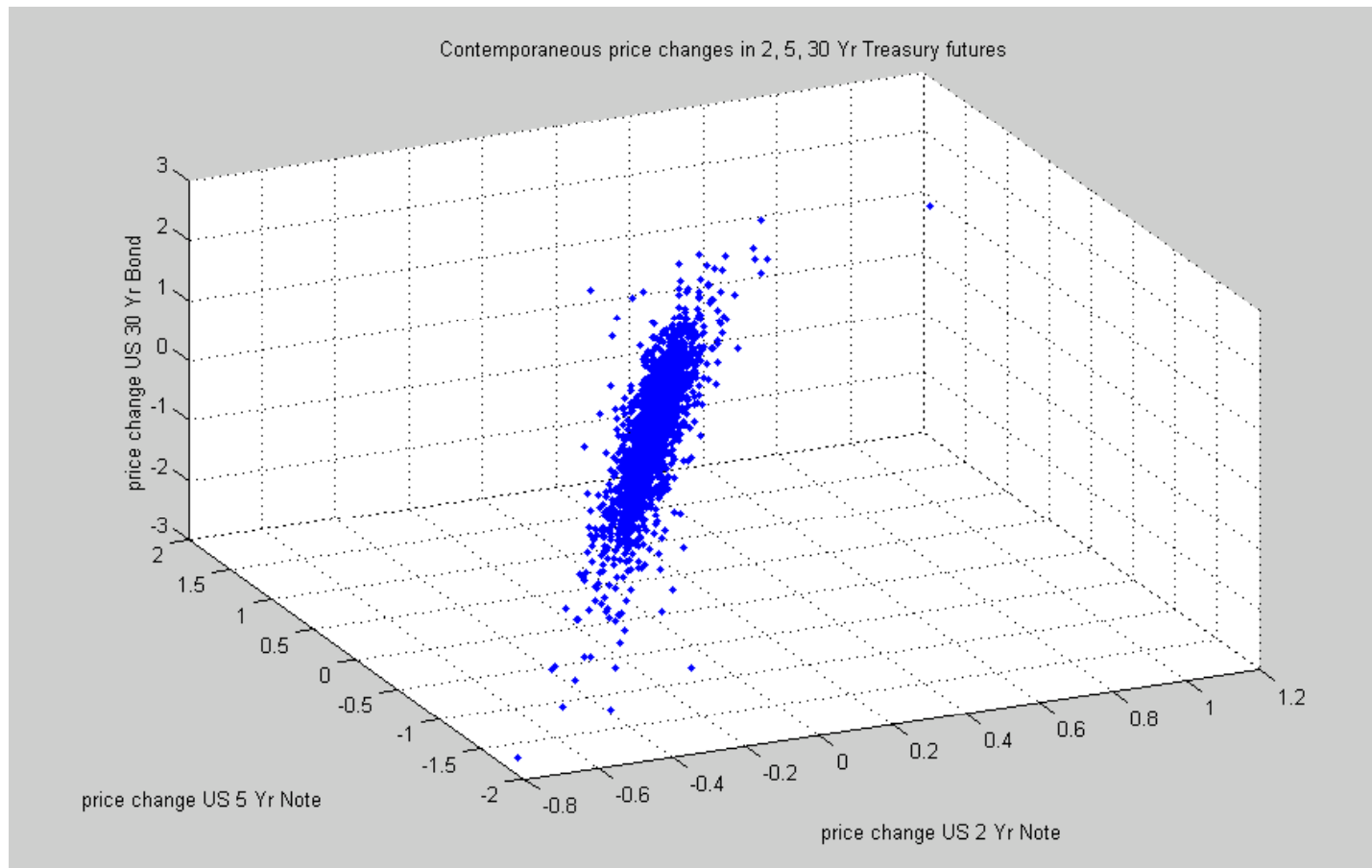
The convexity refers to the difference between the slopes of long-dated versus medium-dated yields and of medium-dated versus short-dated yields.

For example, if 2-year, 5-year, 30-year yields are 1%, 3% and 5% respectively, then the level is  $3 = (1+3+5)/3$ , the slope is  $4 = (5-1)$ , and the convexity is  $0 = (5-3) - (3-1)$ .

## Scatter Swarm of Contemporaneous Price Changes

Here is a 3-dimensional picture showing the contemporaneous close-to-close price changes in the U.S. 2-year, 5-year and 30-year Treasuries futures from 6/25/1990 to 7/15/2003.

It is clear from the cigar shape of the “swarm” that these instruments move together (they are highly positively correlated). Yet they also scatter around their “mean” movement.



## Principal Components

How do we describe changes in tenors which are “mostly” correlated yet have their own smaller idiosyncratic movements outside the “bulk” movement?

We use a standard statistical technique known as *principal components analysis* that decomposes a set of correlated variables into new variables which are linear combinations of the original variables and which are uncorrelated to each other.

Such an analysis of the U.S. 2-year, 5-year and 30-year Treasury futures prices is presented on the next slide. Here we calculated a new set of principal components every 3 months, join together all the 3-month segments and backadjust them over time.

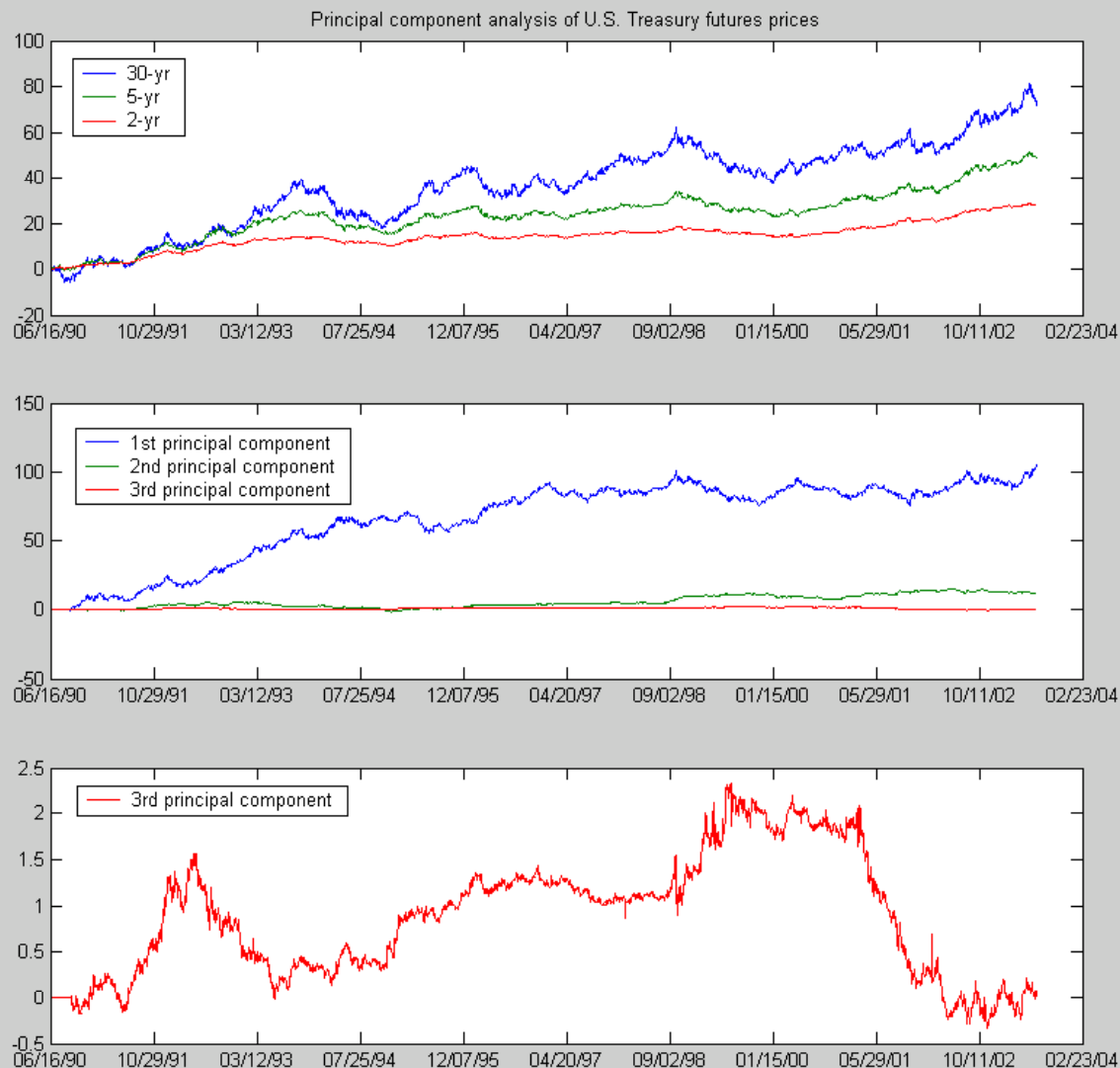
The same analysis of the yields of the theoretical Treasury note or bond underlying the futures is presented on the slide after the next slide.

By construction, the first principal component captures most of the variation in the prices or yields of the three instruments. *It is the most intellectually correct representation of the “general level of interest rates”* (a vague concept which we loosely defined earlier as the average interest rate).

The second principal component captures most of the remaining variation not captured by the first principal component. It is a proxy for the slope of the yield curve.

The third principal component captures most of the remaining variation not captured by the first *and* second principal components. It is a proxy for the convexity of the yield curve.

# Principal Components

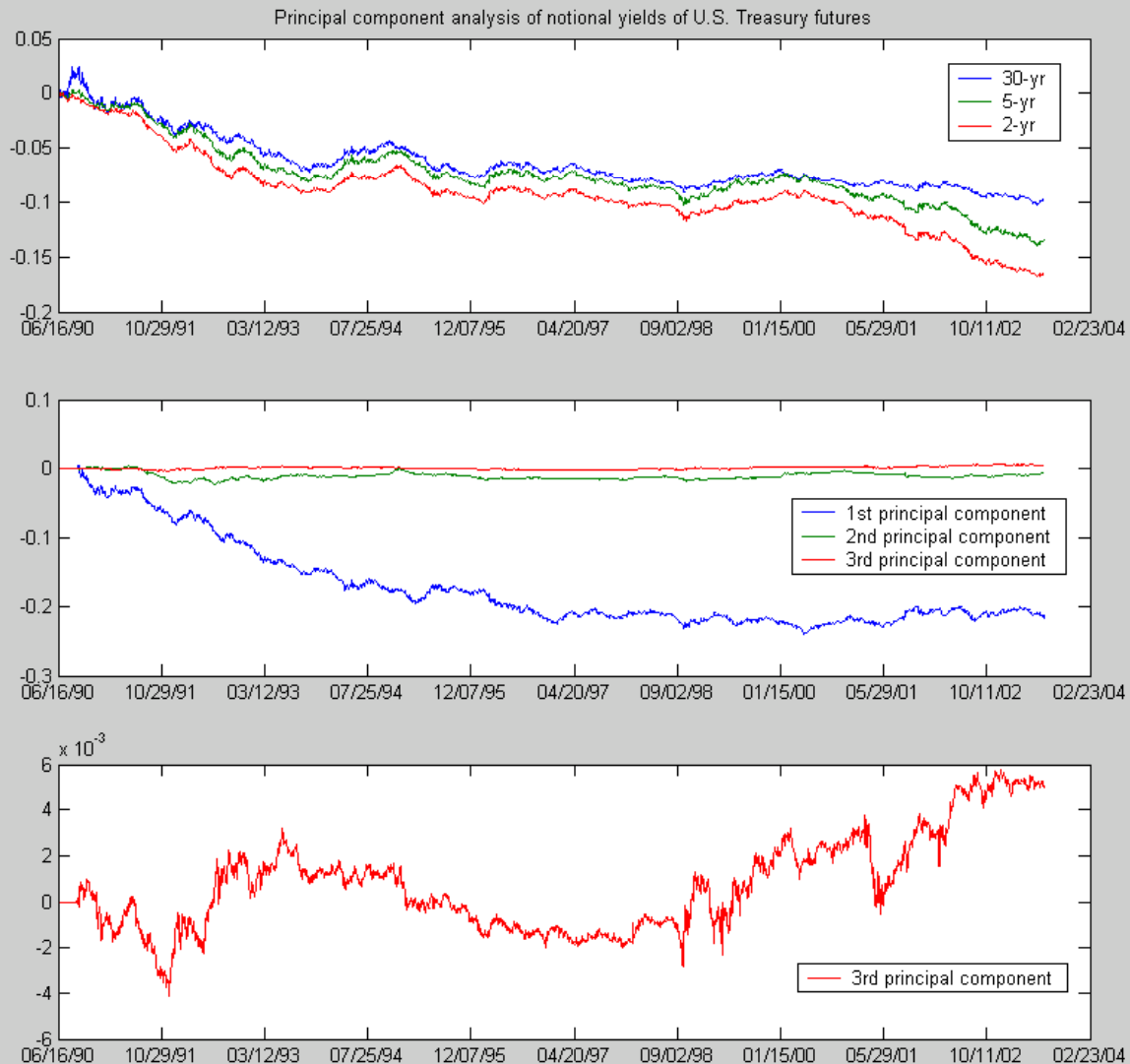


Top panel: Backadjusted prices of 30-yr, 5-yr and 2-yr futures. The prices are referenced to zero at the start of the graph for easier viewing.

Middle panel: Since the principal components are linear combinations of the prices of Treasury futures, they themselves have the same units as the prices.

Third panel: Of the three principal components, the third is the most mean reverting. It has the highest ratio of “fluctuation” relative to “trend”.

# Principal Components



Top panel: Notional yields of 30-yr, 5-yr and 2-yr Treasury futures. The yields are referenced to zero at the start of the graph for easier viewing.

Middle panel: Since the principal components are linear combinations of the notional yields of Treasury futures, they themselves are yields.

Third panel: Of the three principal components, the third is the most mean reverting. It has the highest ratio of “fluctuation” relative to “trend”. However, because of the nonlinear mapping of yield to price, this time series cannot be used directly in historical simulations.



## Trading the 3<sup>rd</sup> Principal Component

The number of contracts of 2 year, 5 year and 30 year futures to trade for each of the principal components will depend on the covariance matrix of the contracts, which varies over time. To illustrate, the number of contracts as of 7/15/03 calculated from the covariance matrix of price changes over the prior 3 months looks like:

	Principal Component		
	1st	2nd	3rd
TU	1	-1	-10
FV	3	-2	6
US	7	1	-1

Thus to reproduce the return corresponding to the third principal component, we need to put on, for every long bond contract, 6 five year contracts and 10 two year contracts. The five year futures will always be in the opposite direction to the two year and long bond futures.

The third principal component can be traded with a simple mean reversion strategy such as a reverse break out system (for example, buying on a 5-day low and selling on a 10-day high). Simulating the performance is straightforward since the component as calculated has the same numeraire (price unit and point value) as the futures.

The performance of a reverse breakout system applied to the third principal component is shown on the next slide. The performance is produced by trading one combination of the three futures markets corresponding to the third principal component shown in the above table.

# Trading the 3<sup>rd</sup> Principal Component

## TradeStation Strategy Performance Report

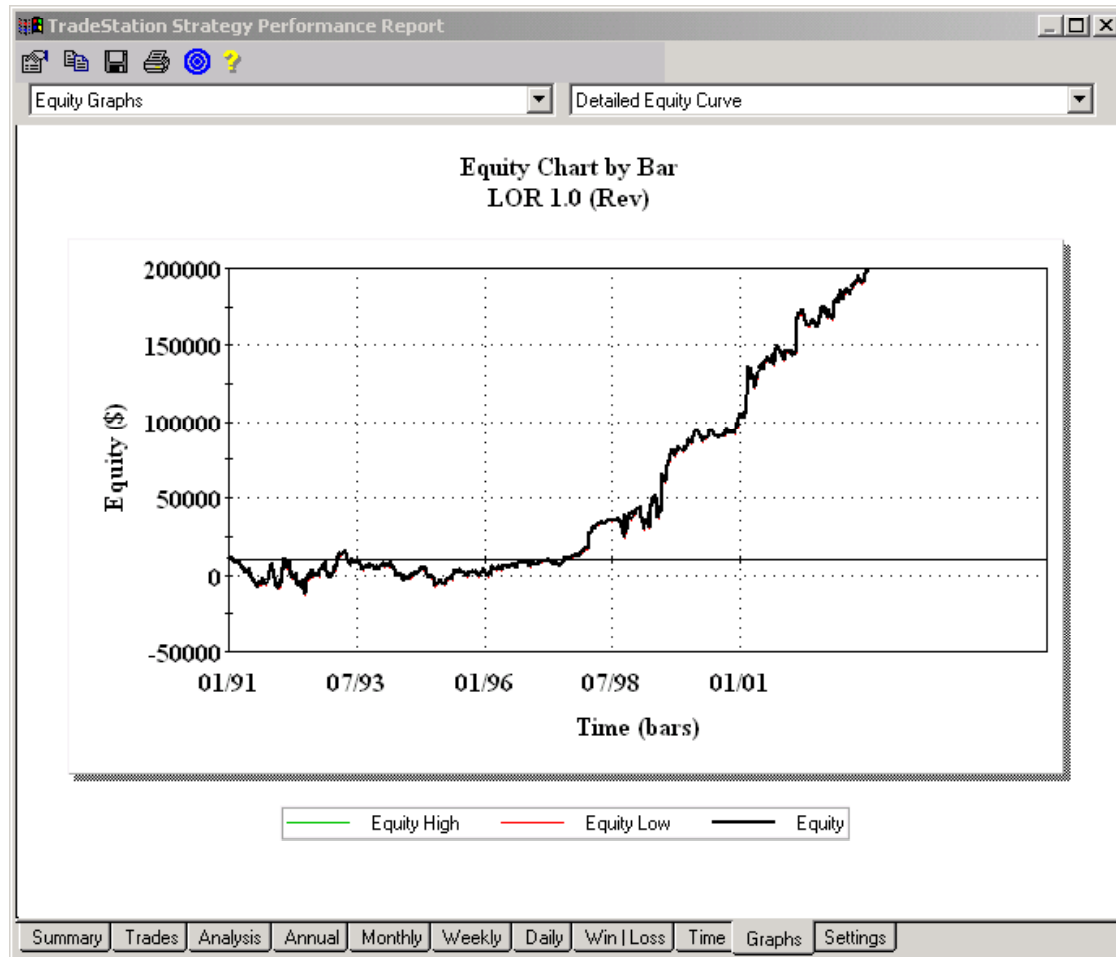
TradeStation Strategy Performance Report - LOR 1.0 (Rev) PCT2530.CSV-Daily (9/10/1990-7/15/2003)

### Performance Summary: All Trades

Total Net Profit	\$187,266.50	Open position P/L	\$0.00
Gross Profit	\$366,881.00	Gross Loss	(\$179,614.50)
Total # of trades	217	Percent profitable	70.97%
Number winning trades	154	Number losing trades	63
Largest winning trade	\$11,848.25	Largest losing trade	(\$11,464.75)
Average winning trade	\$2,382.34	Average losing trade	(\$2,851.02)
Ratio avg win/avg loss	.84	Avg trade (win & loss)	\$862.98
Max consec. Winners	17	Max consec. losers	4
Avg # bars in winners	10	Avg # bars in losers	25
Max intraday drawdown	(\$24,730.00)		
Profit Factor	2.04	Max # contracts held	1
Account size required	\$24,730.00	Return on account	757.24%

# Trading the 3<sup>rd</sup> Principal Component

The cumulated return curve for the reverse break out system applied to the third principal component is shown below. A round turn transaction cost of \$12 per contract is applied in the simulation.



## Trading the 2<sup>nd</sup> Principal Component

The first principal component appears too “trendy”, but the second principal component has enough “fluctuation” relative to “trend” to be traded successfully with a mean reversion system.



# Trading the 2<sup>nd</sup> Principal Component

Shown below is the performance of the system used by my Futures Program applied to the second principal component shown on the previous slide.

## TradeStation Strategy Performance Report

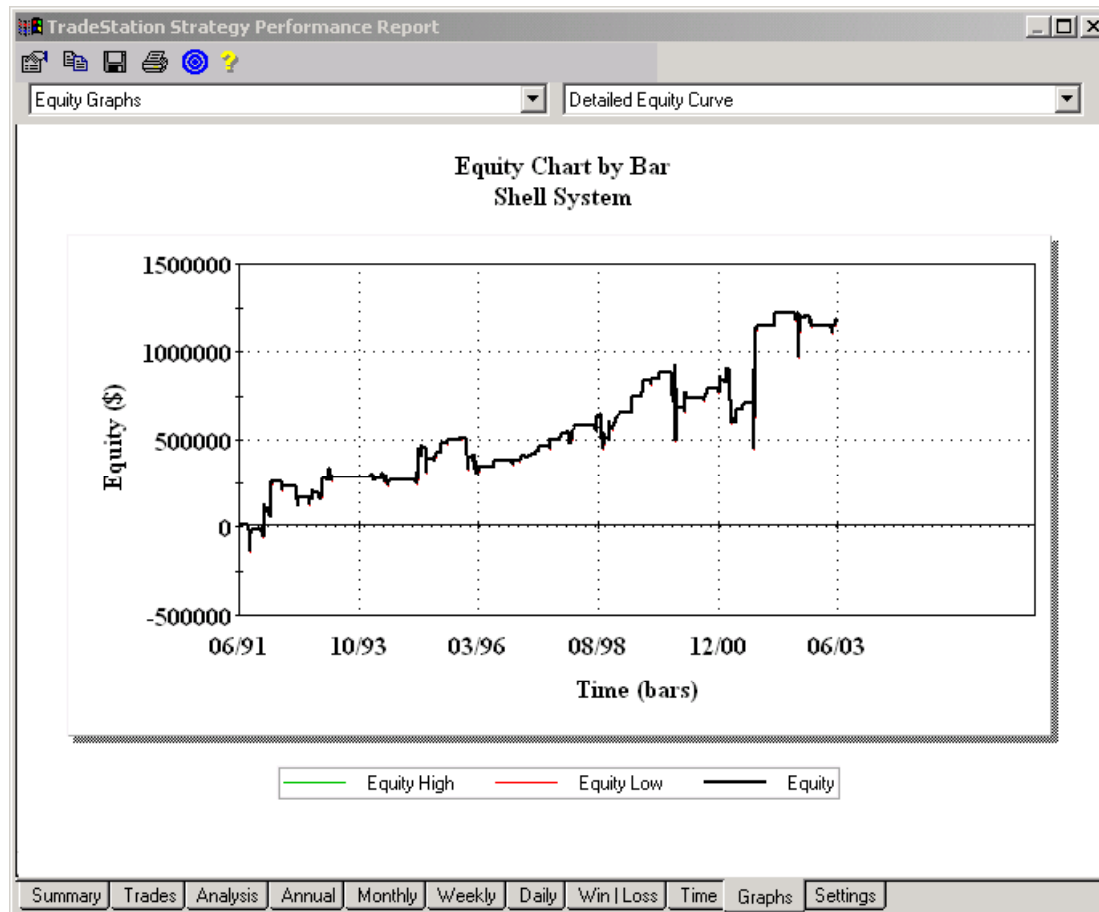
TradeStation Strategy Performance Report - Shell System PCT2\_2~1.CSV-Daily (6/25/1990-7/15/2003)

### Performance Summary: All Trades

Total Net Profit	\$1,161,413.00	Open position P/L	\$0.00
Gross Profit	\$2,958,538.00	Gross Loss	(\$1,797,125.00)
Total # of trades	163	Percent profitable	63.19%
Number winning trades	103	Number losing trades	60
Largest winning trade	\$190,920.00	Largest losing trade	(\$139,280.00)
Average winning trade	\$28,723.67	Average losing trade	(\$29,952.08)
Ratio avg win/avg loss	.96	Avg trade (win & loss)	\$7,125.23
Max consec. Winners	12	Max consec. losers	5
Avg # bars in winners	5	Avg # bars in losers	7
Max intraday drawdown	(\$128,689,616.00)	Max # contracts held	700
Profit Factor	1.65	Return on account	0.90%
Account size required	\$128,689,616.00		

# Trading the 2<sup>nd</sup> Principal Component

The cumulated return curve for the second principal component is shown below. A round turn transaction cost of \$12 per contract is applied in the simulation.



## Conclusion

From a quantitative perspective, mean reversion trading can be described as the search for markets whose “fluctuation” is large relative to “trend”. These markets can be real ones or mathematical constructs arising from the relationships among several markets.

In this monograph, I outlined a standard procedure called principal components analysis with which new “synthetic assets” can be constructed from two or more closely related assets. The analysis is illustrated with U.S. 2 year, 5 year and 30 year Treasury futures.

The historical performance of simple mean reversion strategies applied to the second and third principal components from this analysis shows that they can be successfully traded.

Since the second and third principal components are uncorrelated to the outright markets from which they are constructed, the addition of these components to a portfolio of outright markets will help diversify it.

While I presented only an analysis involving U.S. Treasury futures, the same analysis can be applied to foreign fixed income markets or the swap market (in which case we are trading the corporate yield curve rather than the Treasury yield curve).

The same analysis can also be fruitfully applied to clusters of related stocks.